

# On the Sensitivity of the Polarized Parton Densities to Flavour SU(3) Symmetry Breaking

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## Abstract

Motivated by the possibility that it may be misleading to assume SU(3) symmetry in the analysis of hyperon  $\beta$ -decays, we study the sensitivity of the polarized parton densities, derived from the world data on inclusive polarized deep inelastic lepton-nucleon scattering, to variation of the value of the nonsinglet axial charge  $a_8$  from its SU(3) value of 3F-D.

Our most precise knowledge of the internal partonic structure of the nucleon has come from decades of experiments on unpolarized Deep Inelastic Scattering (DIS) of leptons on nucleons. More recently there has been a dramatic improvement in the quality of the data on *polarized* DIS and consequently an impressive growth in the precision of our knowledge of the polarized parton densities in the nucleon. However, it will be a long time before the polarized data, for the moment limited to neutral current reactions, can match the unpolarized data in volume and accuracy. As a consequence, almost all analyses of the polarized parton densities supplement the DIS (large  $Q^2$ ) data with information stemming from low- $Q^2$  weak interaction reactions. More specifically, it is conventional to use the values of  $G_A/G_V$  from neutron  $\beta$ -decay, and 3F-D from hyperon  $\beta$ -decays to help to pin down the values of the first moments of certain combinations of parton densities:

$$a_3 = (\Delta u + \Delta \bar{u})(Q^2) - (\Delta d + \Delta \bar{d})(Q^2) = \frac{G_A}{G_V}(n \rightarrow p) \equiv g_A , \quad (1)$$

$$a_8 = (\Delta u + \Delta \bar{u})(Q^2) + (\Delta d + \Delta \bar{d})(Q^2) - 2(\Delta s + \Delta \bar{s})(Q^2) = 3F - D , \quad (2)$$

where  $a_3$  and  $a_8$  are the nonsinglet axial charges corresponding to the 3<sup>rd</sup> and 8<sup>th</sup> components of the axial vector Cabibbo current. In (2) F and D are the SU(3) parameters involved in the matrix elements describing hyperon  $\beta$ -decays.

The Bjorken sum rule (1) reflects the isospin SU(2) symmetry, whereas the relation (2) is a consequence of the SU(3) flavour symmetry treatment of the hyperon  $\beta$ -decays. While isospin symmetry is well established, the growing precision of the measurements of magnetic moments and  $G_A/G_V$  ratios in hyperon semi-leptonic decays [1] may be indicating [2]-[4] a non-negligible breakdown of SU(3) flavour symmetry and consequently of Eq. (2). Several attempts have been already made [5, 6] to incorporate some symmetry breaking in the combined analysis of weak interaction data and polarized DIS data.

In this note we present a new study on the sensitivity of the polarized parton densities to SU(3) breaking, using the world data on inclusive DIS [7]. A significant improvement of the previous results on this subject has been achieved. Firstly, a consistent NLO QCD treatment of the data has been carried out. Secondly, unlike the previous analyses, the effects of SU(3) symmetry breaking taken into account in our study are model-independent. Further, we present results on the polarized parton densities *themselves*, not only on their first moments. Finally, we discuss the role of the different factorization schemes.

In the NLO QCD approximation the quark-parton decomposition of the spin-structure function  $g_1(x, Q^2)$  has the following form:

$$g_1(x, Q^2) = \frac{1}{2} \sum_q^{N_f} e_q^2 [(\Delta q + \Delta \bar{q}) \otimes (1 + \frac{\alpha_s(Q^2)}{2\pi} \delta C_q) + \frac{\alpha_s(Q^2)}{2\pi} \Delta G \otimes \delta C_G], \quad (3)$$

where  $\Delta q(x, Q^2)$ ,  $\Delta \bar{q}(x, Q^2)$  and  $\Delta G(x, Q^2)$  are quark, anti-quark and gluon polarized densities which evolve in  $Q^2$  according to the spin-dependent NLO DGLAP equations [8]. In (3)  $\delta C_{q,G}$  are the NLO terms in the spin-dependent Wilson coefficient functions and the symbol  $\otimes$  denotes the usual convolution in Bjorken  $x$  space.  $N_f$  is the number of flavours.

According to QCD, perfect data on the independent structure functions  $g_1^p$  and  $g_1^n$  (or  $g_1^d$ ) *uniquely* determine the individual parton densities  $(\Delta q + \Delta \bar{q})(x, Q^2)$  and  $\Delta G(x, Q^2)$  in the DIS region. It is the difference of their  $Q^2$  evolution that enables them to be extracted from the data separately. However, bearing in mind the limited range in  $x$  and  $Q^2$  and the accuracy of the present data, it is immediately clear that the separation of the polarized parton densities from each other will not be very clear-cut. In other words, the unknown free parameters attached to the input parton densities at some arbitrary  $Q^2 = Q_0^2$  are correlated and not well determined from the fit to the inclusive DIS data alone. (Note that the attempts [9] to extract the polarized quark densities from *semi-inclusive* data are not entirely successful because, due to the quality of these data at present, many additional assumptions had to be made.) That is why our knowledge on the first moments of the polarized parton densities coming from the hyperon semi-leptonic decays (Eqs. (1) and (2)) is usually used in addition to fix some of the free parameters. It should be noted that the nonsinglet axial charges  $a_3$  and  $a_8$  are  $Q^2$  independent and this is the fact which helps us to link the information from both high and low- $Q^2$  regions.

We have performed an NLO QCD fit to the world data on  $g_1^N(x, Q^2)$  [7] using for the first moments of the quark densities the relations (1) and (2). All details of our approach are given in [10, 11]. In contrast to our previous analyses [10]-[12], where we always used for  $a_8$  its SU(3) symmetric value [13]  $3F - D = 0.579 \pm 0.025$ , we incorporate now in our study the effects of the symmetry breaking. These effects have been taken into account in a model-independent way, i.e. we have used for  $a_8$  different fixed values in the interval [0.36, 0.86]. These values cover the range obtained in various attempts to construct theoretical models of SU(3) breaking.

The results of the analysis are presented in the JET scheme (see [11] and references therein). The independence of the physical results on the choice of the factorization has been demonstrated in our previous papers [11, 12]. A remarkable property of the JET scheme is that the first moment of the singlet quark density,  $\Delta\Sigma(Q^2)$ , as well as  $(\Delta s + \Delta\bar{s})(Q^2)$ , the first moment of the strange sea quarks in the nucleon, are  $Q^2$  independent quantities. Then, in the JET scheme is it meaningful to directly interpret  $\Delta\Sigma$  as the contribution of the quark spins to the nucleon spin and to compare its value obtained from DIS region with the predictions of the different (constituent, chiral, etc.) quark models at low  $Q^2$ .

It is useful to recall the transformation rules relating  $\Delta\Sigma(Q^2)$  and  $(\Delta s + \Delta\bar{s})(Q^2)$  in the JET and  $\overline{\text{MS}}$  schemes, the latter having been used in the previous papers [6] accounting for the SU(3) breaking effects:

$$\Delta\Sigma_{\text{JET}} = \Delta\Sigma_{\overline{\text{MS}}}(Q^2) + N_f \frac{\alpha_s(Q^2)}{2\pi} \Delta G(Q^2) , \quad (4)$$

$$(\Delta s + \Delta\bar{s})_{\text{JET}} = (\Delta s + \Delta\bar{s})_{\overline{\text{MS}}}(Q^2) + \frac{\alpha_s(Q^2)}{2\pi} \Delta G(Q^2) , \quad (5)$$

where  $\Delta G(Q^2)$  is the first moment of the polarized gluon density  $\Delta G(x, Q^2)$  (note that  $\Delta G$  is the same in the factorization schemes under consideration).

It is important to mention that the difference between the values of  $(\Delta s + \Delta\bar{s})$ , obtained in the  $\overline{\text{MS}}$  and JET schemes could be large due to the axial anomaly. To illustrate how large it can be, we present the values of  $(\Delta s + \Delta\bar{s})$  at  $Q^2 = 1 \text{ GeV}^2$  obtained in our recent analysis [12] of the world DIS data in the  $\overline{\text{MS}}$  and JET schemes (the SU(3) limit for  $a_8$  has been used):

$$(\Delta s + \Delta\bar{s})_{\overline{\text{MS}}} = -0.10 \pm 0.01, \quad (\Delta s + \Delta\bar{s})_{\text{JET}} = -0.06 \pm 0.01 . \quad (6)$$

Let us now comment briefly on how the deviation of  $a_8$  from its SU(3) symmetric value influences the results on the polarized parton densities extracted from the DIS data. In order to demonstrate the sensitivity of the parton densities to the SU(3) breaking we present them (Figures 1-5) and their first moments (Table 1) for three typical values of  $a_8$ . All results are given at  $Q^2 = 1 \text{ GeV}^2$  ( $N_f = 3$ ). One can see from Table 1 that the values of  $\chi^2$  are practically insensitive to the change of  $a_8$ , which means that any of the solutions presented in this paper cannot be excluded by the present data. One can also conclude that except for the strange sea quarks and the gluons the other densities are essentially those determined by the SU(3) analysis of the data. It is important to stress that the singlet quark density, as well as its first moment,  $\Delta\Sigma$  (the spin of the nucleon carried by the quarks), are virtually unchanged

by the SU(3) breaking. The mean value of  $\Delta\Sigma$  ranges from 0.34 to 0.40 and within the errors is not far from the value 0.60 expected in low- $Q^2$  quark models (see, e.g., [14]).

**Table 1.** Sensitivity of the first moments of the polarized parton densities to SU(3) symmetry flavour symmetry breaking ( $Q^2 = 1 \text{ GeV}^2$ , DOF=155). The SU(3) value 3F-D=0.58.

$a_8$	$\chi^2$	$\Delta u + \Delta \bar{u}$	$-(\Delta d + \Delta \bar{d})$	$-(\Delta s + \Delta \bar{s})$	$\Delta\Sigma$	$\Delta G$
0.40	127.4	$0.81 \pm 0.02$	$0.45 \pm 0.02$	$0.02 \pm 0.01$	$0.34 \pm 0.05$	$0.13 \pm 0.14$
3F-D	128.3	$0.86 \pm 0.02$	$0.40 \pm 0.02$	$0.06 \pm 0.01$	$0.40 \pm 0.04$	$0.57 \pm 0.14$
0.86	127.4	$0.90 \pm 0.02$	$0.35 \pm 0.02$	$0.15 \pm 0.02$	$0.40 \pm 0.06$	$0.84 \pm 0.30$

Contrary to the singlet and non-strange quarks, the strange sea polarization changes significantly when flavour SU(3) symmetry is broken (see Fig. 3). Comparing with the SU(3) case the strange sea contribution to the nucleon spin is reduced by a factor of three when  $a_8 = 0.40$  and enhanced by a factor of two and a half when  $a_8 = 0.86$ . In the case  $a_8 = 0.40$  the strange polarization is consistent with zero in agreement with the usual assumption in low- $Q^2$  quark models. This fact, contrary to what is sometimes claimed (see, e.g., [15]), does not help to solve the "spin crisis in the naive parton model" since the value of  $\Delta\Sigma$  remains virtually unchanged. The mean value of the gluon polarization  $\Delta G$  ranges from 0.13 to 0.84, but because of the large errors these values are consistent within two standard deviations. As seen from Table 1, although a significant improvement of the quality of the data since the EMC experiment has been achieved, the present data still do not exclude a vanishing gluon polarization in the DIS region if the value of  $a_8$  is considerably smaller than the SU(3) value.

It is important to emphasize that although the axial charge  $a_8$  and the strange sea quarks ( $\Delta s + \Delta \bar{s}$ ) cannot be well separated using the current DIS data alone,  $a_8$  and ( $\Delta s + \Delta \bar{s}$ ) are *independent* quantities ( $\Delta q_8(x, Q^2)$  and  $(\Delta s + \Delta \bar{s})(x, Q^2)$  evolve with  $Q^2$  in different ways). That is why any combined analysis of the DIS and the hyperon  $\beta$ -decays data, in which the issue of the SU(3) breaking is model-dependent, has to take account of this fact. Otherwise, such an analysis is inconsistent, as was already discussed in detail in our note [16].

In conclusion, we note that a further improvement of the situation is expected to come from: *i)* the current KTeV experiment at Fermilab on the  $\Xi^0$   $\beta$ -decay,  $\Xi^0 \rightarrow \Sigma^+ e \bar{\nu}$ , (see [15] and references therein) and *ii)* a combined analysis of inclusive and semi-

inclusive present and future DIS data. While the KTeV experiment will help in clarifying the issue of the SU(3) breaking, and thereby to find the proper value of  $a_8$ , the future DIS experiments will be very important for a precise determination of the polarized parton densities (and, in particular,  $a_8$ ) *independently* of the information coming from the low- $Q^2$  region. The consistency of the values of  $a_8$  obtained from the high and low- $Q^2$  regions will be a good test for our understanding of the spin properties of the nucleon.

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## Figure Captions

**Fig. 1.** Next-to-leading order polarized parton density  $x(\Delta u + \Delta \bar{u})$  at  $Q^2 = 1 \text{ GeV}^2$  (JET scheme). The solid, short-dashed and long-dashed curves correspond to the values 0.58 (SU(3) limit), 0.86 and 0.40 for the axial charge  $a_8$ , respectively. The error band accounts for the statistical and systematic uncertainties.

**Fig. 2.** Next-to-leading order polarized parton density  $x(\Delta d + \Delta \bar{d})$  at  $Q^2 = 1 \text{ GeV}^2$  (JET scheme). See caption to Fig. 1.

**Fig. 3.** Next-to-leading order polarized strange sea quark density  $x(\Delta s + \Delta \bar{s})$  at  $Q^2 = 1 \text{ GeV}^2$  (JET scheme). See caption to Fig. 1.

**Fig. 4.** Next-to-leading order polarized singlet quark density  $x\Delta\Sigma$  at  $Q^2 = 1 \text{ GeV}^2$  (JET scheme). See caption to Fig. 1.

**Fig. 5.** Next-to-leading order polarized gluon density  $x\Delta G$  at  $Q^2 = 1 \text{ GeV}^2$  (JET scheme). See caption to Fig. 1.

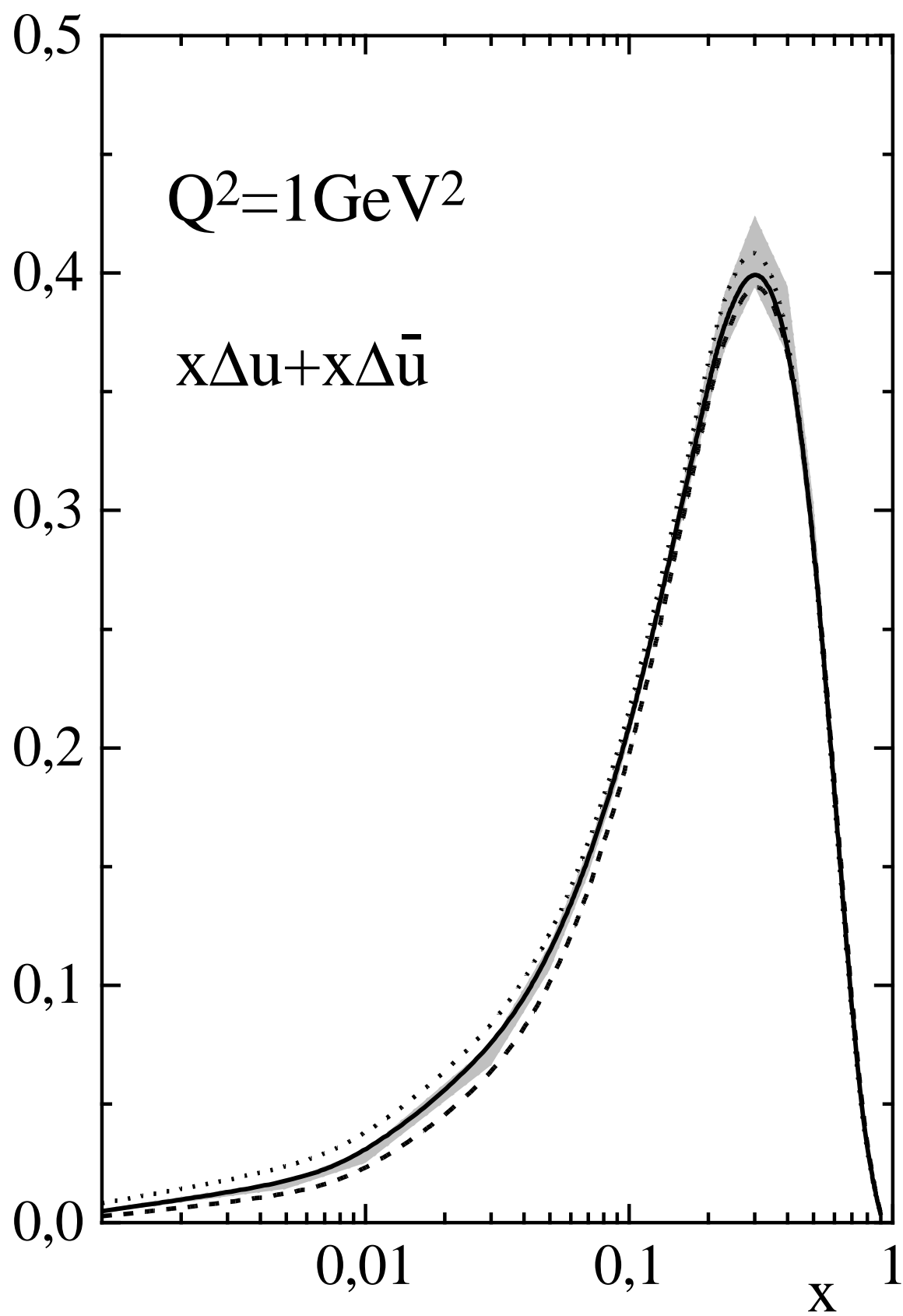


Fig. 1

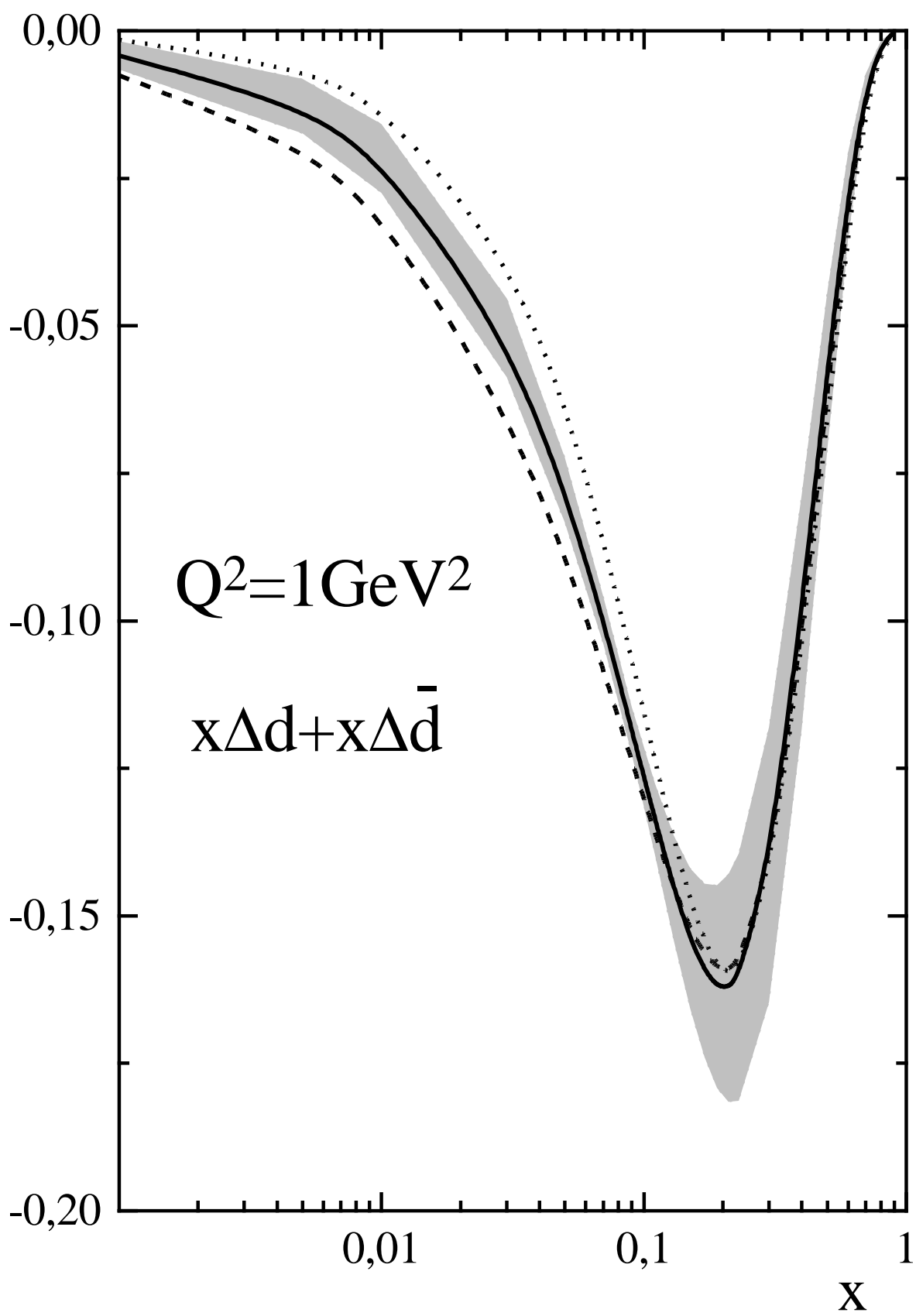


Fig. 2

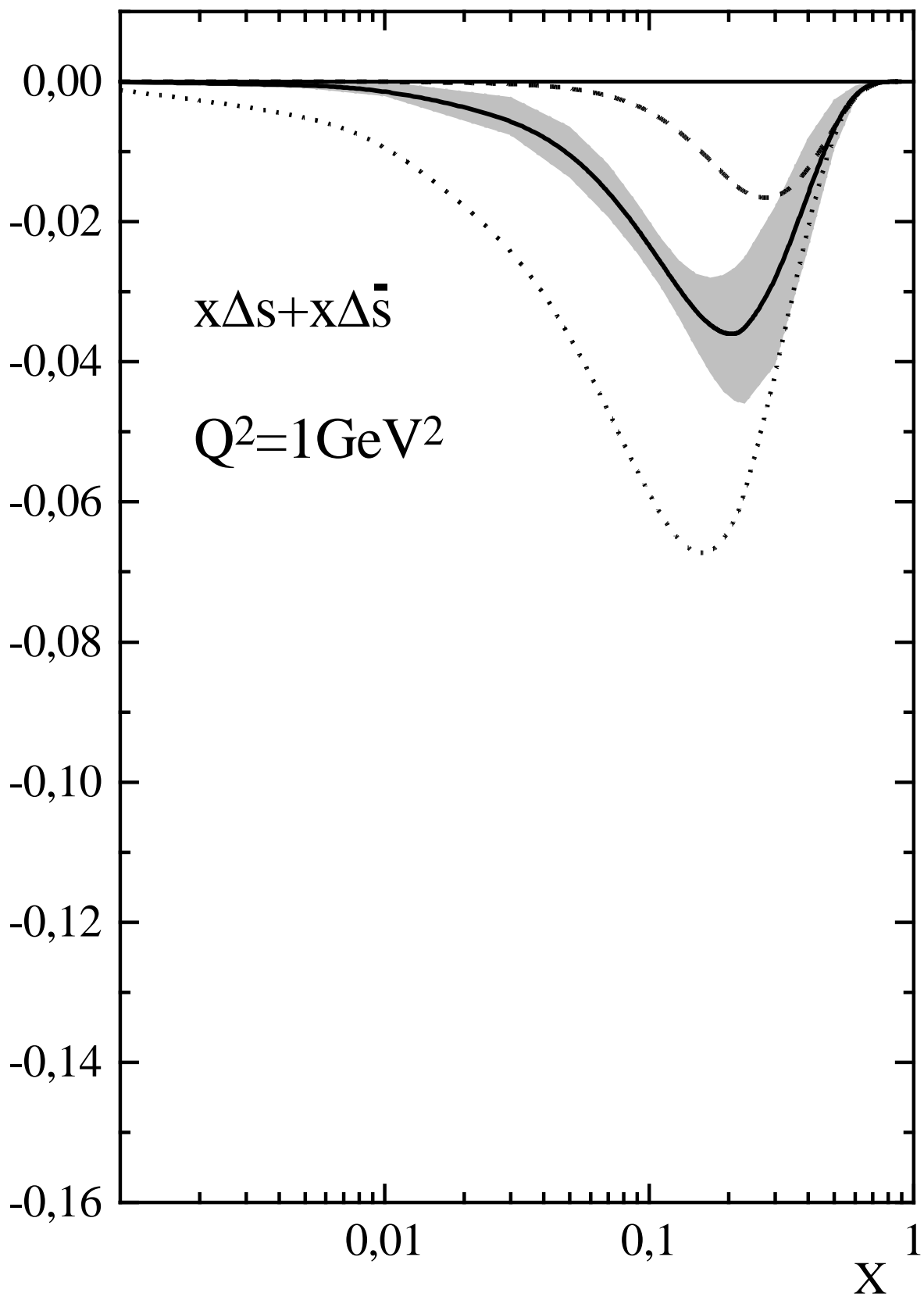


Fig. 3

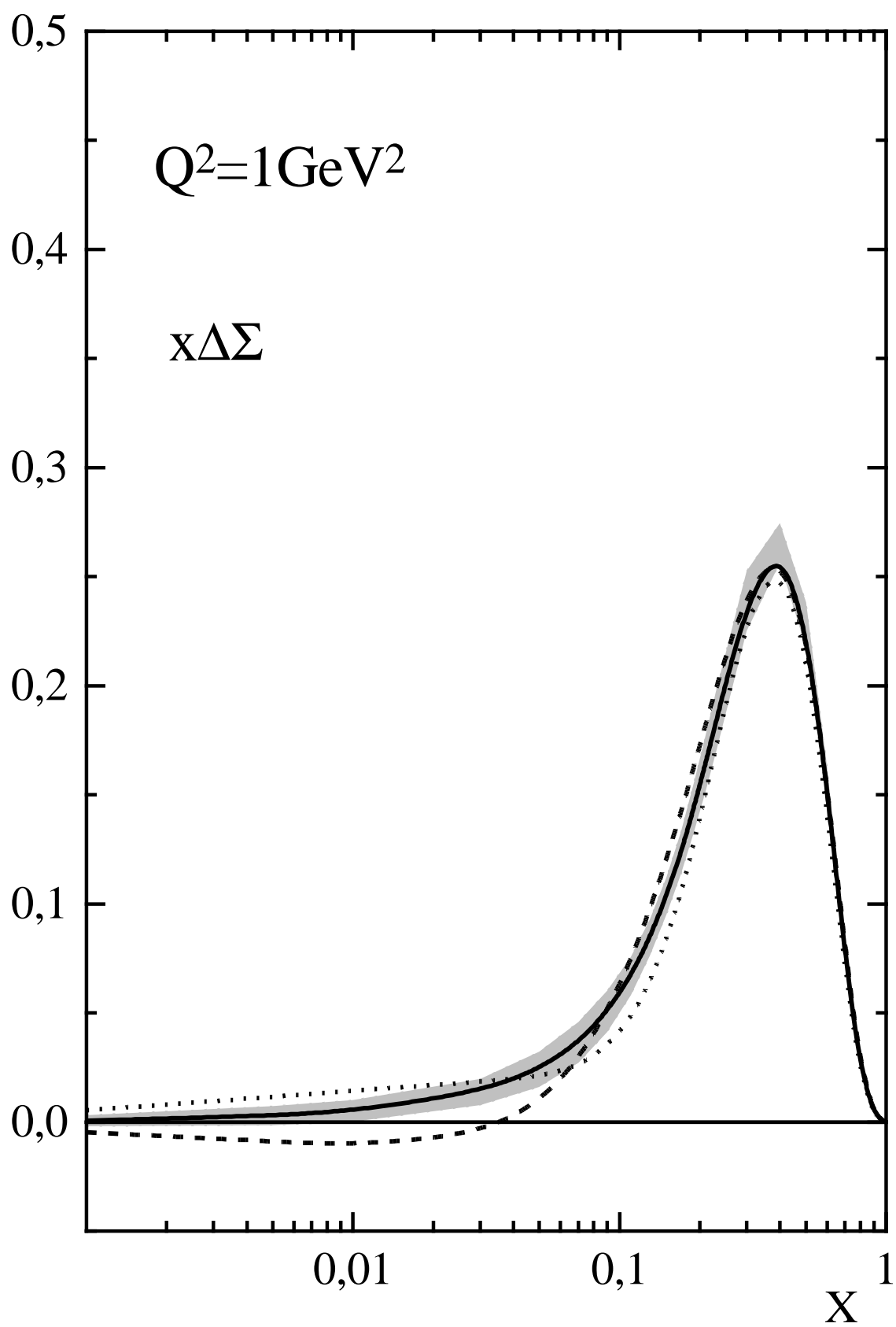


Fig. 4

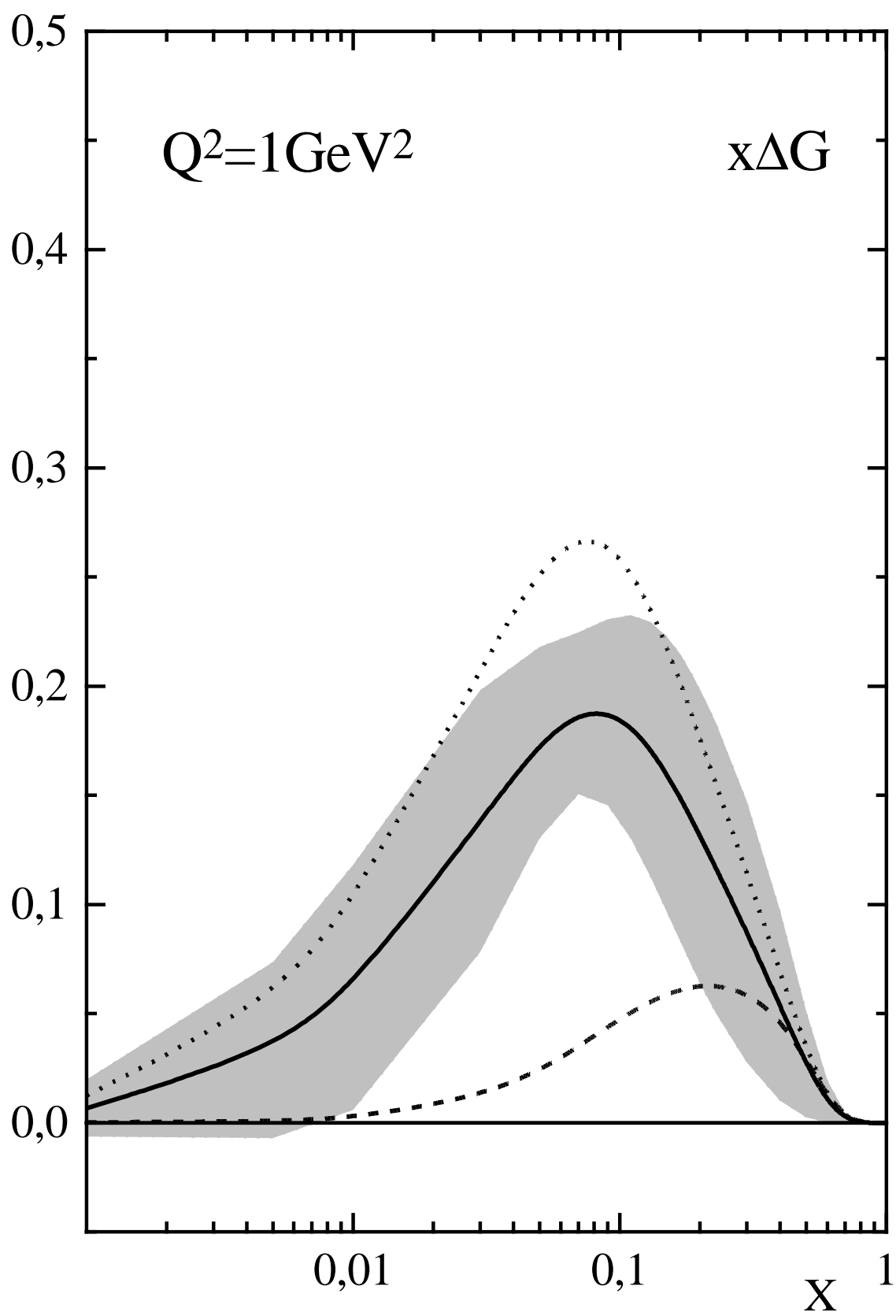


Fig. 5